# Astroparticle Physics, NAASPH-12

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Exam, April 4, 2014; 14:00-17:00; Room EB 5412.0025
5 problems (total of 55 points).
Write the solution of every problem on a separate piece of paper with name and student number.

**Write clearly, it should be readable.**

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**Problem 1** (16 pnts in total)

3 pnts a. Name three different acceleration mechanisms of charged particles by cosmic objects and indicate how they differ from each other.

3 pnts b. Which of these three acceleration mechanisms is considered to be the most effective one to accelerate charged particles to energies beyond $10^{12}$ eV.

5 pnts c. In Figure 1 you see the extragalactic Galaxy Centaurus-A in X-ray, optical and radio emission. This Galaxy is on the top of the list as accelerators to energies beyond $10^{20}$ eV. The center of this Galaxy hosts a super massive black-hole, which emits jets extending far beyond matter as observed through optical telescopes. These jets are visible as the structures going to the upper left and to the lower right in this figure, especially in continuous radio emission (plot labeled as "NRAO Radio continuum"). Describe where you would expect that acceleration of charged particles would be possible and use some of the objects / structures in this figure to substantiate your hypothesis about possible sites for acceleration.

5 pnts d. The distance between Earth and Centaurus-A is about 4 MPc. Use the plot in Figure 2 to explain whether or not it is likely that protons with an energy beyond $10^{20}$ eV can reach Earth for the case that the distance is 4 MPc. The same for a distance of 400 MPc. Explain your answers.

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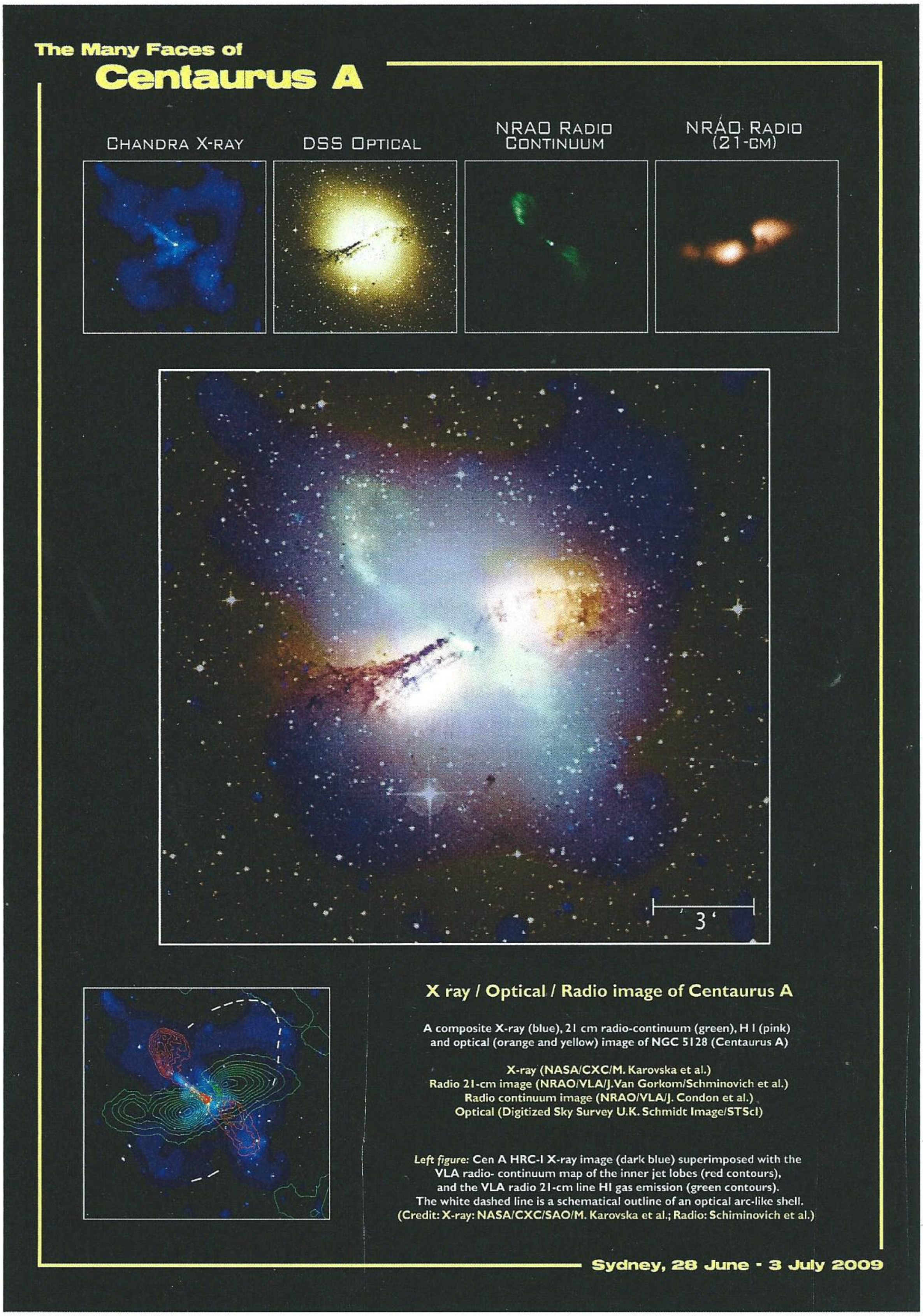



Figure 1: Extragalactic object Cen-A in different wave lengths.

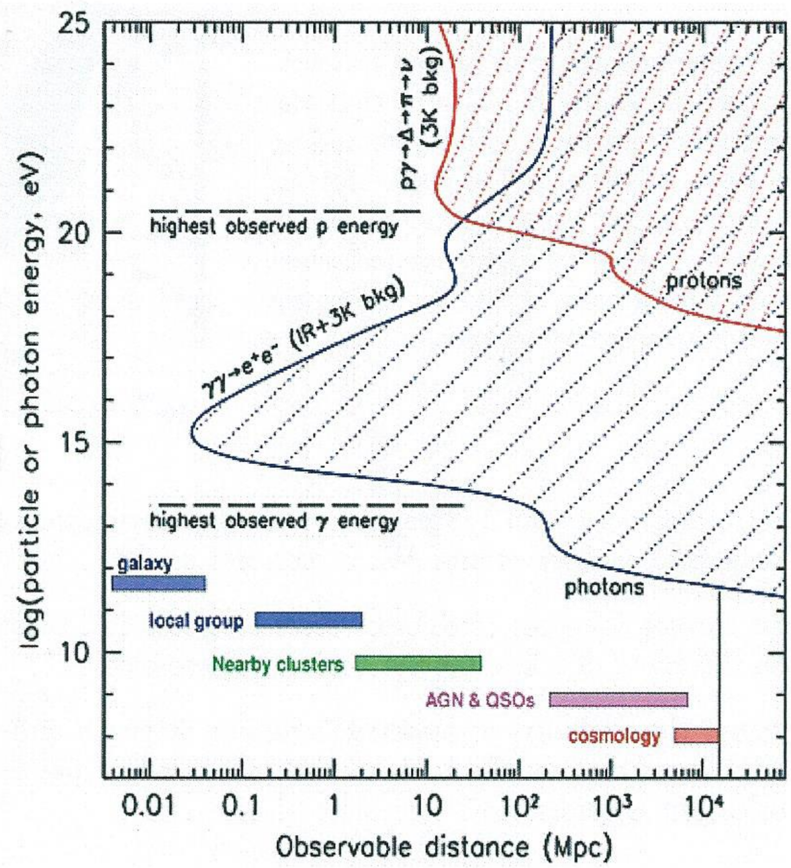



Figure 2: Proton and photon horizons as function of traveled distance and primary energy.

**Problem 2** (9 pnts in total)

3 pnts a. List three different mechanisms for producing high-energy ($E > 10^{10}$ eV) gamma-rays. Explain, which physics environments are required for each of these mechanisms.

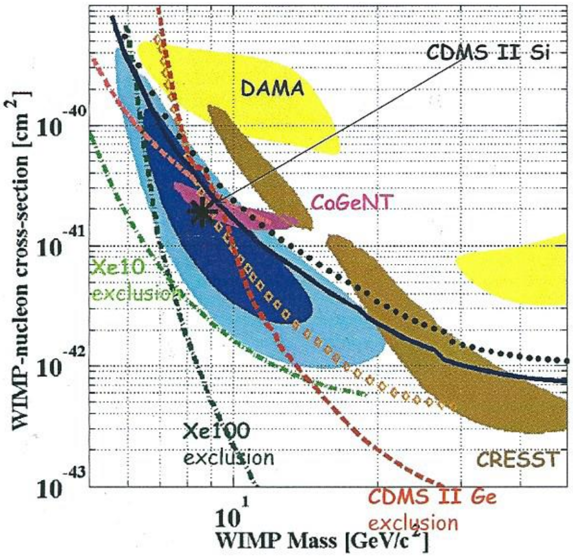
2 pnts b. Describe how high-energy gamma-rays are being detected when their energy is larger than about $10^{10}$ eV.

2 pnts c. List two different mechanisms how high-energy ($E > 10^{10}$ eV) neutrinos can be produced. Explain, which physics environments are required for each of these two mechanisms.

2 pnts d. Describe how high-energy neutrinos are being detected when their energy is larger than about $10^{10}$ eV.

**Problem 3** (13 pnts in total)

Dark Matter has not yet been proven to exist. However, there are many indications that the energy density in the Universe today should be attributed for more than 20% to Dark Matter. Recently evidence has been found that Dark Matter may be formed by right-handed, sterile, neutrinos with a mass of 7 keV. The figure shows a well-known exclusion plot.




4 pnts a. Describe (in not more than 1/3 page) how Zwicky determined evidence for Dark Matter from Coma-cluster observations. What data did he use?

4 pnts b. Show that the recoil energy (for elastic scattering in a NaI crystal) will be well below the detection threshold of 2 keV for the DAMA experiment.

3 pnts c. In the figure the excluded or preferred coupling strength and mass for DM particles is indicated. Argue that on the basis of this plot a 7 keV particle with a cross section of $10^{-40}\,\mathrm{cm}^2$ is not excluded.

2 pnts d. Describe (in not more than 1/4 page) the experimental observation which lies at the basis of the 7 keV right-handed neutrino.

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**Problem 4** (10 pnts in total)

When an ultra-high-energy proton arrives at the top of the atmosphere a particle cascade (air shower) develops.

2 pnts a. Give a very short (2 or 3 lines) explanation why initially the energy in the electromagnetic part (electrons, positrons, and photons) of the air shower increases at the expense of the energy in the hadronic part.

3 pnts b. Assume that in the cascade at a height of 10 km a charged pion ($\pi^+$) of energy $10^{14}\,\mathrm{eV}$ is produced. Give the mean distance (in g/cm$^2$) for the pion to decay (neglecting interactions) and compare this with the mean distance for the particle to interact (neglecting decay). Do the same for a neutral pion ($\pi^0$). Use the values given on the last page of the exam.

5 pnts c. Assume a $\pi^0$ of energy $10^{14}\,\mathrm{eV}$ is created at a penetration depth $X_1$ and starts an electromagnetic shower. Calculate the penetration depth $X_{max}$ (in g/cm$^2$) where the shower reaches its maximum and the number of particles at this maximum. Take $\lambda_{em} = 26\,\mathrm{g/cm}^2$ (the particle doubling distance) and $E_c = 10^8\,\mathrm{eV}$ (the energy after which doubling stops, rounded off for simplicity).

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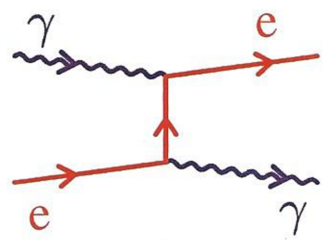
**Problem 5** (7 pnts in total)

The process of Compton scattering off a low-energy electron is described by the Feynman diagram to the right. Here we will use it to calculate the matrix element for this process. The high-energy (real) photon has an energy $E_\gamma = E$ and $\vec{q} = E$ in the $\hat{z}$-direction while the electron should be taken at rest initially. Time runs from left to right.




Assume that the $\hat{z}$-component of the momenta of out going electron, $p'_z$, and photon, $q'_z$, are equal and that each has a small component in the $\hat{x}$-direction.

1 pnts a. Show that the $\hat{x}$-components of the momenta are equal and opposite, $p'_x = -q'_x$. Show also that $p'_z = E/2 = q'_z$.

1 pnts b. Show that the energy of the outgoing photon can be expressed as $E'_\gamma = E/2 + (p'_x)^2/E$ for $p'_x \ll E$.

1 pnts c. Show that the energy of the outgoing electron can be expressed as $E'_e = E/2 + (p'_x)^2/E + m_e^2/E$ for $p'_x \ll E$.

1 pnts d. Show that $(p'_x)^2 = m_e E/2$ for $m_e \ll E$.

3 pnts e. Give the expression for the momentum and the propagator of the intermediate electron. Show that it is proportional to $1/E$.

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**Some numbers**

Electron mass $m_e c^2 = 511\,\text{keV}$; Muon mass $m_\mu c^2 = 106\,\text{MeV}$; Pion mass $m_\pi c^2 = 140\,\text{MeV}$;
Proton mass: $m_p c^2 = 0.938\,\text{GeV}$
Conversion: $1\,\text{eV}/\text{c}^2 = 1.78 \times 10^{-36}\,\text{kg}$
Boltzmann's constant: $k = 8.62 \times 10^{-11}\,\text{MeV/K}$
Planck's constant: $h = 4.1 \times 10^{-15}\,\text{eV s}$
Avogadro's number: $N_A = 6 \times 10^{23}\,/\text{mol}$
Solar Mass: $\text{M}_\odot = 1.99 \times 10^{30}\,\text{kg}$
Parsec: $1\,\text{pc} = 3.1 \times 10^{16}\,\text{m}$
Velocity of Sun w.r.t. center Milkyway: $V_\odot = 270\,\text{km/s}$
Velocity of Earth in orbit around the Sun: $V_\oplus = 30\,\text{km/s}$
Typical galactic dark matter density: $\rho_{DM} = 9\ \text{k}\ \text{M}_\odot/\text{pc}^3$

**Air-shower physics**

At 10 km height the density of the atmosphere is $0.4 \times 10^{-3}\,\text{g}\,\text{cm}^{-3}$.
The penetration depth for pions in air is $\lambda_\pi = 120\,\text{g}\,\text{cm}^{-2}$, for protons $\lambda_p = 90\,\text{g}\,\text{cm}^{-2}$, and for iron is $\lambda_{Fe} = 5\,\text{g}\,\text{cm}^{-2}$.
The mean travel distance in vacuum of a pion with energy $E$ is $d_{\pi^0} = \gamma 25 \times 10^{-9}\,\text{m}$ and $d_{\pi^\pm} = \gamma 7.8\,\text{m}$ where the relativistic $\gamma$-factor is given by $\gamma = E/mc^2$ and $m_\pi c^2 = 140\,\text{MeV}$.

**Integrals**

For $c > 0$ we have:

$$\int_0^\infty c\,e^{-cx} dx = 1\,; \quad \int_0^\infty c\,x\,e^{-cx} dx = 1/c\,; \quad \int_0^\infty c x^2 e^{-cx} dx = 2/c^2$$